Dilution Of Precision in Acoustics

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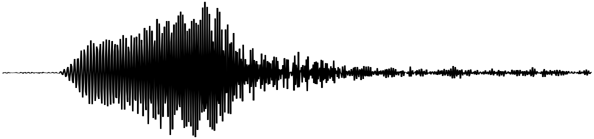
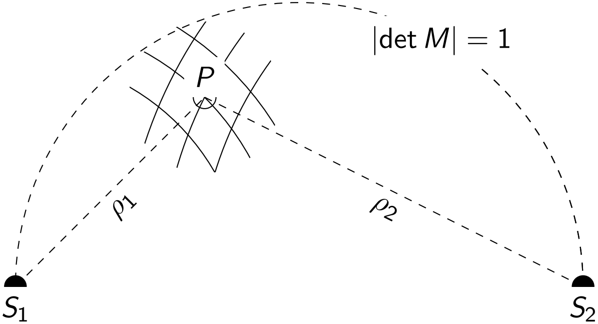
DOP i Akustikk



Feel free to ask questions now, this evening (!), and otherwise.



Abstract

The concept of Dilution Of Precision (DOP) is central in the analysis of the performance of the satelite based Global Positioning System (GPS). It can also be used in other contexts, and in particular within the corresponding acoustic positioning problem. The underlying theory is highly relevant also in a variety of other acoustical problems. Several examples from acoustics will be mentioned.

Contents

Global Positioning System (GPS)

Illustration courtesy of NASA from http://www.gps.gov.

Global Positioning System (GPS)

The precision obtained from multiple satellites in view of a receiver combine according to the relative position of the satellites to determine the level of precision in each dimension of the receiver measurement.

This is quantified with the DOP factor k_d.

When visible GPS satellites are close together in the sky the DOP factor is high and when far apart the DOP factor is low.

For simplicity we neglect that also the time coordinate is estimated by GPS Chaffee and Abel [1994], Langley [1999].

Dilution Of Precision (DOP)

The standard deviation σ of the position estimate is given by

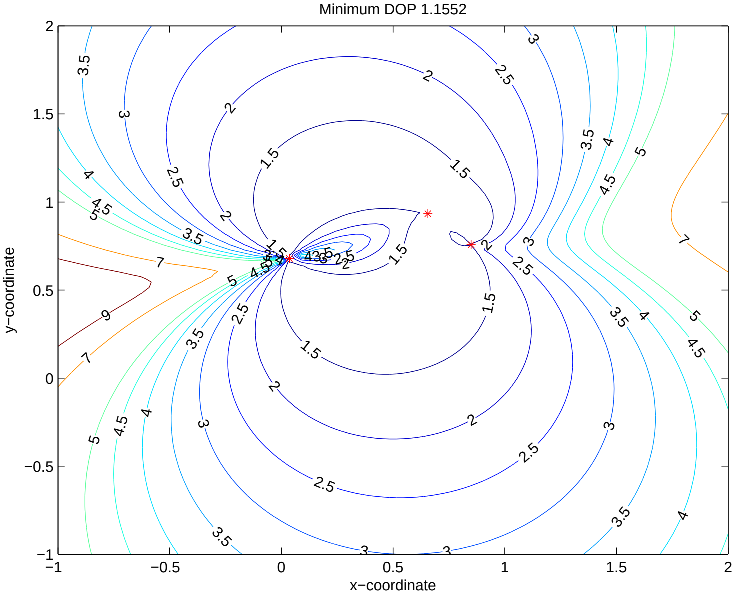
$$\sigma = k_d \sigma_R \tag{1}$$

where σ_R is the assumed standard deviation of the range measurements and the DOP factor k_d is

$$k_d = \sqrt{\mathrm{tr}\left[(M^T M)^{-1}\right]} \tag{2}$$

The rows in the matrix M are given by the unit vectors pointing from the actual position towards the satellites.

Geometry of DOP factor k_d in two dimensions



Sources of errors in range (σ_R)

$$\sigma_R = \sqrt{3^2 + 5^2 + 2.5^2 + 2^2 + 1^2 + 0.5^2}\mathrm{m} = 6.7\mathrm{m}$$

Wikipedia

Source	Effect (m)
Signal arrival C/A	±3
Signal arrival P(Y)	±0.3
Ionospheric effects	±5
Ephemeris errors	±2.5
Satellite clock errors	±2
Multipath distortion	±1
Tropospheric effects	±0.5
σ_R C/A	±6.7
σ_R P(Y)	±6.0

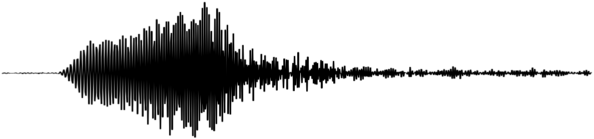
- Choice of signal for signal delay estimation is important: C/A and P(Y) codes for GPS.
- The ionosphere and troposphere causes dispersion and signal speed variations.
- These errors have corresponding errors in for instance Underwater Positioning Systems (UPS) and in Real Time Locating Systems (RTLS).

Summary so far

- Position accuracy σ is given by distance accuracy σ_R and DOP factor k_d:

$$\sigma = k_d \sigma_R$$

- Distance accuracy σ_R depends on many factors. Choice of signal for delay estimation is one important factor.



- The DOP factor k_d is given by the position and the choice of geometry for the satellites = anchor nodes = beacons:

$$k_d = \sqrt{\operatorname{tr}(M^T M)^{-1}}$$

Underwater Positioning System (UPS)

- Underwater positioning systems can be used and are used for many purposes: Personal GPS, Remotely Operated underwater Vehicles (ROVs), Autonomous Underwater Vehicles (AUVs), acoustic fish tags, ...
- An array of beacons located near the seafloor, near the surface, or distributed otherwise to give an underwater personal GPS is one possibility.
- The concept of DOP can be used, and has been used, as presented above also for UPS.
- There are many other possible systems based on combinations of active, passive, one-way, or two-way signaling. The important problem of tracking is related, but will not be considered.
- In the following the discussion is restricted to systems that estimate the position based on range measurements from several known positions.

A personal UPS

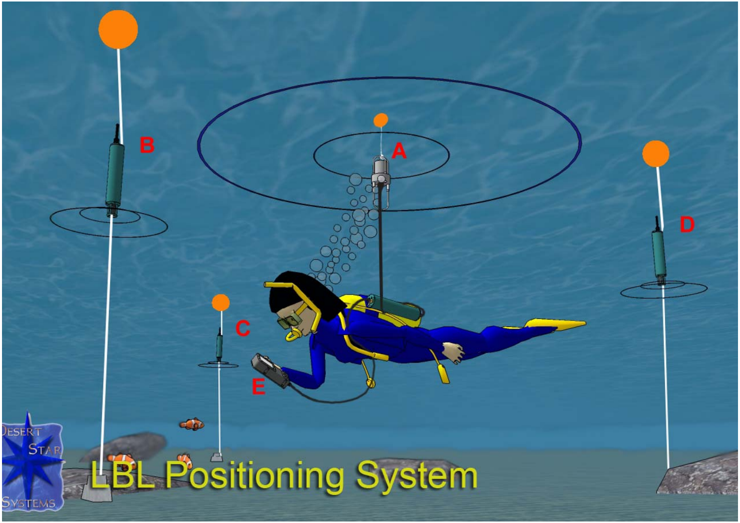


Illustration courtesy of Desert Star Systems LLC from Wikipedia.

Autonomous Underwater Vehicle (AUV)

Illustration courtesy of Bluefin Robotics, International Submarine Engineering Ltd. from Wikipedia.

Statistical model

- The observed distance $\boldsymbol{\rho}_i$ to beacon number i can be modeled by

$$\boldsymbol{\rho}_i = R_i + \sigma_R \mathbf{n}_i \tag{3}$$

where R_i is the true unknown distance, and σ_R is assumed to be a known constant.
- $\sigma_R \mathbf{n}_i$ represents the sum of range errors as described above.
- **The standard model** is given by assuming that the $\mathbf{n}_i$ are independent Gaussian variables with zero mean and unit variance.

DOP for GPS

- For the standard model it follows from the Cramer-Rao theorem that equation (1) gives a lower bound on the standard deviation of *any unbiased* estimator of the position.
- In the GPS case equation (3) can be linearized and the lower bound equals the standard deviation of the maximum likelihood estimator (MLE) for all practical purposes, and the MLE is essentially unbiased and optimal.

DOP for UPS

- For the standard model it follows from the Cramer-Rao theorem that equation (1) gives a lower bound on the standard deviation of *any unbiased* estimator of the position.
- In the UPS case equation (3) can be linearized, but this is seldom a good approximation.
- The lower bound does not equal the standard deviation of the maximum likelihood estimator.
- The MLE is biased and sub-optimal.
- **New results**:
  - There exist alternative unbiased estimators.
  - The MLE can be improved.

The linear model

- Linearization gives a model equivalent to the standard linear regression model

$$\mathbf{y} = X\beta + \sigma_e \mathbf{n}$$

  with $X = M$, $\sigma_e = \sigma_R$, and the regression coefficients β corresponds to the position.
- The optimal unbiased estimator is

$$\hat{\boldsymbol{\beta}} = (X^* X)^{-1} X^* \mathbf{y}$$

- The covariance is

$$\operatorname{Cov} \hat{\boldsymbol{\beta}} = \sigma_e^2 (X^* X)^{-1}$$

  and the trace gives the formula (2) for the geometrical dilution of precision k_d.

The Cramer-Rao bound

- Let T be an unbiased estimator of $\tau(\theta)$, so

$$\mathrm{E}\, T = \tau$$

- The Cramer-Rao or information inequality is then

$$\mathrm{Cov}\, T \geq (\partial_\theta \tau)^* I^{-1} \partial_\theta \tau$$

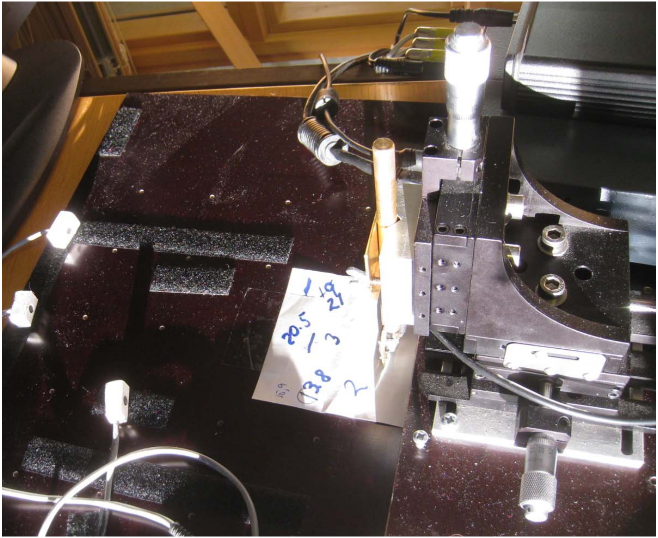
- The Fisher information matrix is given by

$$I = \mathrm{E}\left(\partial_\theta \log f(X \mid \theta)(\partial_\theta \log f(X \mid \theta))^*\right)$$

- $X \sim N(\mu(\theta), \Sigma)$ gives

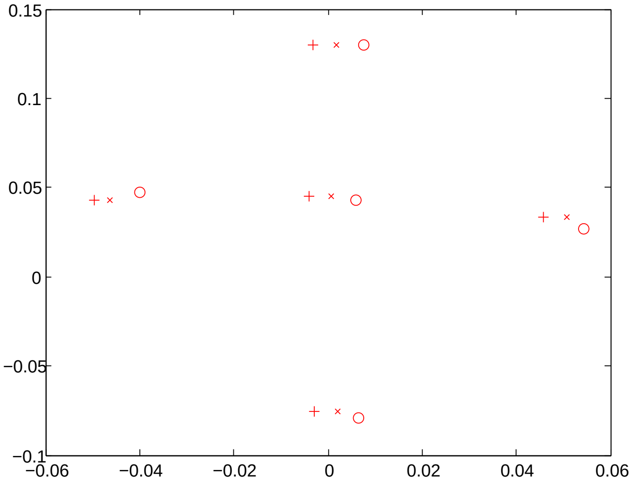
$$I = (\partial_\theta \mu)^* \Sigma^{-1} (\partial_\theta \mu)$$

Ultrasound positioning



Experiments with Tone Berg and Tor Arne Reinen.

Ultrasound positioning

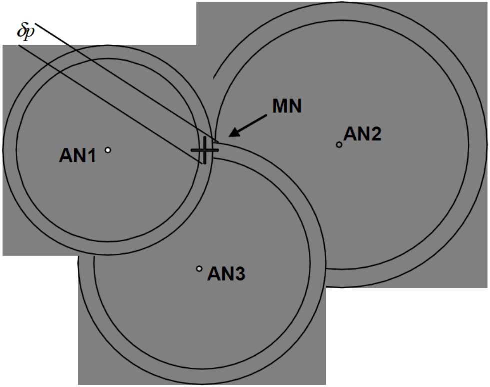


Experiments with Tone Berg and Tor Arne Reinen.

DOP for Real Time Locating Systems (RTLS IEEE)

- Real Time Locating Systems (RTLS), are systems that provide the location of assets on a constant and recurrent basis.
- According to the International Standards Organization (ISO/IEC 24730) an RTLS is a set of radio frequency receivers and associated computing equipment used to determine the position of a transmitting device relative to the placement of the aforementioned receivers.
- RTLS typically refers to systems that provide automatic determination of location information.

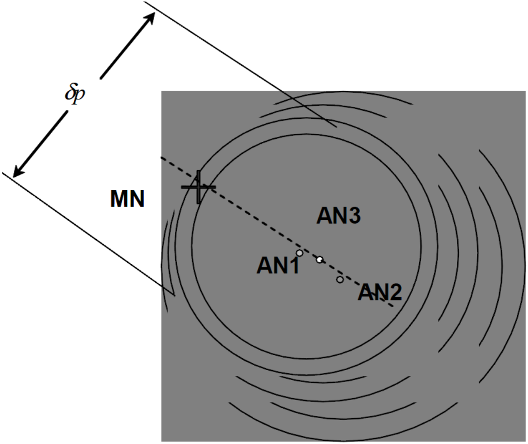
DOP for Real Time Locating Systems (RTLS IEEE)



MN = mobile node; AN = anchor node

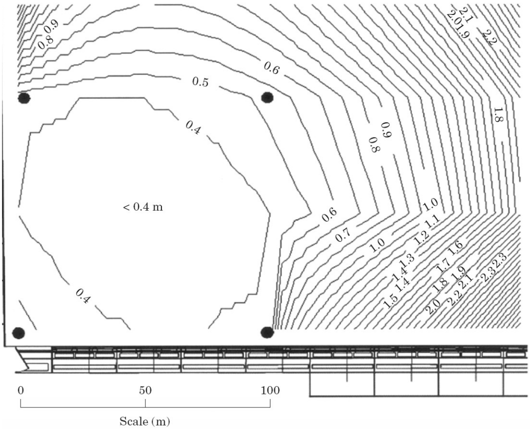
Good DOP case, from IEEE 802.15.4a-2007.

DOP for Real Time Locating Systems (RTLS IEEE)



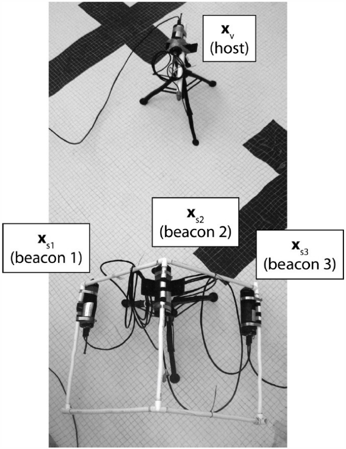
Bad DOP case, from IEEE 802.15.4a-2007.

DOP for fish tag systems [Ehrenberg and Steig, 2002]



Plan view of the dam and hydrophone locations showing a contour plot of position errors for a depth of $z = 49\mathrm{m}$.

DOP for underwater vehicles [Bingham, 2009]



The three beacons in the lower part of the image are fixed to a frame where the baselines are all $0.59\mathrm{m}$. The host is located at a variable distance away from the beacon network. Shown here the host is approximately $2\mathrm{m}$ from the beacons.

Sound power of road vehicles [Taraldsen, 2004]

The method estimates the expected value of the sound power of a population of vehicles. Furthermore the method estimates the weight of the sub-sources in the vehicle model and the ground impedance of the road surface. The method is based on statistical modelling and gives in particular estimates of the expanded uncertainty (ISO GUM) for all of the above mentioned parameters.

Ground effect and impedance

A recently published one-parameter ground model based on Darcy's law is here generalized into a two-parameter model which depends on an effective flow resistivity and an effective layer depth. Extensive field measurements of the acoustic impedance of various ground types have been carriedout for frequencies in the range from 200 Hz to 2.5 kHz.

Useful links from the WEB

http://www.citeulike.org/user/gtaralds/tag/gt_iassta

http://www.gps.gov

http://freegeographytools.com/2007/
determining-local-gps-satellite-geometry-effects-on-positi

http://en.wikipedia.org/wiki/Gps

http://en.wikipedia.org/wiki/Error_analysis_for_the_
Global_Positioning_System

http://en.wikipedia.org/wiki/Real_time_locating

http://en.wikipedia.org/wiki/Chirp_spread_spectrum

http://standards.ieee.org/getieee802/download/802.15.
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J. Chaffee and J. Abel. GDOP and the Cramer-Rao bound. In *Position Location and Navigation Symposium*, pages 663–668. IEEE, 1994. doi: 10.1109/PLANS.1994.303374. URL http://dx.doi.org/10.1109/PLANS.1994.303374.

John E. Ehrenberg and Tracey W. Steig. A method for estimating the "position accuracy" of acoustic fish tags. *ICES J. Mar. Sci.*, 59(1):140–149, January 2002. doi: 10.1006/jmsc.2001.1138. URL http://dx.doi.org/10.1006/jmsc.2001.1138.

R. B. Langley. Dilution of Precision. *GPS World*, 1999. URL http://gauss.gge.unb.ca/papers.pdf/gpsworld.may99.pdf.

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Summary

- **The concept of DOP from GPS is most useful also for UPS.**
- Alternative statistical models and estimators should be considered for UPS, and for other acoustic positioning systems.
- The statistical modeling and in particular the use of the Cramer-Rao lower bound have many other possible applications in acoustics and otherwise.
- **The ML estimator for GPS position is biased, but there exist unbiased alternative estimators.**
- Thank You for your attention!